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PREFACE

Modern Problems in Optics and Photonics: Proceedings of the International Advanced Research Workshop (Armenia, 27 August - 2 September, 2009). - Yerevan, 2009 - 130 p. The proceedings contain the papers presented at the International Advanced Research Workshop on "Modern Problems in Optics and Photonics", MPOP-2009. The Workshop was held in Armenia, from 27 August to 2 September, 2009 (http://www.ipr.sci.am/mpop/). Hosts were Yerevan State University and Institute for Physical Research of National Academy of Sciences. The Workshop was dedicated to the 90-th Anniversary of the Yerevan State University. It provided an opportunity to bring together experienced and young scientists on various facets of modern optics and photonics, to converse and consider the most recent developments in these areas. The plenary talks at the Workshop have been separately published in "Modern Optics and Photonics: Atoms and Structured Media", World Scientific Publishing Co.(http://www.worldscientific.com). The present volume is the second part of the Proceedings and includes oral talks and posters. All of the papers have been reviewed by the Editors and Publishing Council of NAS (session of Council, April 5, 2010), all works represent the state of the art of research in this area. We would like to express our sincere gratitude to the authors of the papers. We also thank the sponsors of the Workshop: State Committee of Science of Armenia, National Foundation of Science and Advanced Technologies (NFSAT), International Society of Optics Engineering (SPIE), Optical Society of Armenia (OSA), LT-PYRKAL cjsc. We would like to mention the valuable role of members of the Organizing and Program Committees.

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First Demkov-Kunike Model in Cold Molecule Formation: Large Detuning Regime

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We analyze a two-term ansatz recently proposed to describe the quadratic-nonlinear version of the level-crossing Demkov-Kunike problem relevant to cold molecule formation via coherent photo- or magneto-association of ultracold atoms. Analyzing the large detuning regime which in terms of photoassociation describes the behavior of the system in cases when the detuning variation range and the resonance sweep rate are large simultaneously, we show that it is subdivided into three distinct interaction regimes - weak, intermediate and strong coupling limits. We show that the ansatz accurately describes the whole temporal dynamics of the association process in all the mentioned interaction regimes. We show that in contrast to the Landau-Zener model it is impossible to convert all the atoms of the system into molecules whatever the pulse area. This limitation on the conversion efficiency can be interpreted as a nonlinear saturation.

1. Introduction

We study the nonlinear dynamics of cold molecule formation in an atomic Bose-Einstein condensate[1] for the first Demkov-Kunike (DK) quasi-linear level-crossing model.[2] This model is known from the linear theory of nonadiabatic transitions and is considered as a natural physical generalization of the Landau-Zener model.[3] The weak interaction limit of the nonlinear DK problem, corresponding to small laser field intensity, has been discussed in Refs.,[4,5] where analytic formulas for the final probability of transition to the molecular state have been proposed applying different approaches. On the other hand, the strong interaction limit of the DK model, corresponding to high laser field intensity, has been studied in Ref.[6,7] where two distinct scenarios of evolution are shown to be available – the small and large detuning regimes. The small detuning regime describes the behavior of the system when both the detuning variation range and the resonance sweep rate are small whereas the large detuning regime represents the opposite situation.

In the present paper, we discuss a variational ansatz[8] which accurately describes the association process in the large detuning regime for both weak and strong coupling limits. In addition, this solution well describes the intermediate regime of moderate coupling too. The approximation not only provides an analytical formula for the final transition probability to the molecular state but also accurately describes the whole time dynamics of the coupled atom-molecular system.

We consider the molecule formation process via

coherent photo- or magneto-association of ultracold atoms in an atomic Bose-Einstein condensate[1] described in the mean-field approximation by the following nonlinear two-state problem

$$\begin{aligned} i\tfrac{da_1}{dt} &= U(t)e^{-i\delta(t)}\bar{a}_1 a_2, \\ i\tfrac{da_2}{dt} &= \tfrac{U(t)}{2}e^{i\delta(t)}a_1 a_1, \end{aligned} \tag{1}$$

where a_1 and a_2 are the atomic and molecular states' probability amplitudes, respectively, $U(t)$ is referred to as the Rabi frequency (for definiteness, we use the photoassociation terminology), and $\delta(t)$ is the integral of the detuning of the field frequency from the frequency of the transition. Equation (1) indicates that $|a_1|^2 + 2|a_2|^2 = 1$. The quantities $|a_1|^2$ and $|a_2|^2$ are the fractions of atoms and molecules, respectively, with respect to the initial number of atoms in the system. Hereafter we refer to $p_1 = |a_1|^2$ as the atomic state probability and to $p = |a_2|^2$, conventionally, as the molecular state probability (note that $p_1 \in [0,\ 1]$ whereas $p \in [0,\ 1/2]$). We consider the initial conditions $a_1(-\infty) = 1$ and $a_2(-\infty) = 0$. Finally, we discuss the field configuration given by the first Demkov-Kunike (DK) level-crossing model of a bell-shaped pulse defined as:[2]

$$U = U_0 \mathrm{sech}(t), \quad \delta_t = 2\delta_0 \tanh(t). \tag{2}$$

This problem has been discussed in Refs.,[4–9] where analytic formulas for the final probability of transition to the molecular state have been proposed for different interaction limits. It has been shown that two distinct scenarios of evolution are possible – the small and large detuning regimes.[6,7] The main peculiarity of the molecule association process in the small detuning regime is its pronounced oscillatory behavior while in the large detuning regime only weak, damped oscillations between the atomic and molecular subsystems occur.

2. Mathematical treatment

Here, we discuss the large detuning regime ($\delta_0 \geq 1$) which models a situation when the laser, being far detuned in the beginning of the interaction passes, linearly in time, through the resonance, and eventually becomes again far detuned. To describe this regime quantitatively we apply the (exact) equation for the molecular state probability $p = |a_2|^2$[6]

$$\left(\frac{d}{dz} - \frac{\delta^*_{zz}}{\delta^*_z}\right)\left[p_{zz} - \frac{\lambda}{2}\left(1 - 8p + 12p^2\right)\right] + \delta^{*2}_z p_z = 0, \tag{3}$$

where

$$\begin{aligned} &\delta^*_z = 2\delta_0 \tan(z), \\ &z(t) = 2\mathrm{Arctan}(e^t) - \tfrac{\pi}{2}, z \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right). \end{aligned} \tag{4}$$

We show that in the large detuning regime under consideration a highly accurate approximation describing the whole time evolution of the molecular state probability can be written as a sum of the solution $p_0(A,t)$ of an *augmented limit* equation[8] and a scaled solution to the *linear* DK problem[2] with *modified* parameters $P_{DK}(\lambda^*, \delta_0, t)$:

$$p = p_0(A,t) + C^* \frac{P_{DK}(\lambda^*, \delta_0, t)}{P_{DK}(\lambda^*, \delta_0, \infty)}. \tag{5}$$

Here, A, C^*, and λ^* are variational parameters and the function $P_{DK}(\lambda^*, \delta_0, \infty)$ is the final transition probability for the associated *linear* problem given by the known formula[2]

$$P_{DK}(\lambda^*, \delta_0, \infty) = 1 - \cos^2\left(\pi\sqrt{\lambda^* - \delta_0^2}\right)\ \mathrm{sec}\, h^2 \pi\delta_0. \tag{6}$$

The proposed ansatz involves three variational parameters, A, C^*, and λ^*. We now examine the behavior of these parameters in the limit of large field amplitudes (strong coupling limit), $\lambda \to \infty$. To do this, we substitute the trial function (7) into the exact equation for the molecular state probability (5) and examine the behavior of the remainder

$$\begin{aligned} \frac{R}{C^*} = \left(\frac{d}{dz} - 2\csc(2z)\right) \times \\ \left(4\left[\lambda(1 - 3p_0) - \lambda^*\right]\frac{P_{DK}(\lambda^*, z(t))}{P_{DK}(\lambda^*, z(+\infty))} + \right. \\ \frac{2\lambda^*}{P_{DK}(\lambda^*, z(+\infty))} + \left(\frac{p_{0zz} - A}{C^*}\right) - \\ \left. 6\lambda_C^* \frac{P^2_{DK}(\lambda^*, z(t))}{P^2_{DK}(\lambda^*, z(+\infty))}\right);. \end{aligned} \tag{7}$$

The examination of the limit $t \to +\infty$ then suggests that the parameters C^* and λ^* are approximately expressed in terms of the parameter A as follows:[9]

$$\lambda^* = \lambda\left(1 - 3p_0(+\infty)\right), p_0(+\infty) = \frac{1}{2} - \sqrt{\frac{A}{2\lambda}} \tag{8}$$

and

$$C^* = \sqrt{\frac{A}{6\lambda}}. \tag{9}$$

Hence, we see that the principal variational parameter of ansatz (5) is A. Hence, we try to construct a general expression for this parameter. The asymptotic behavior of function A at small and large limiting values of the parameter λ is known.[5,9] Now, we try to construct a function that has the given asymptotes. A possible version is the following smooth function:

$$A = \left(\frac{\delta_0}{\sqrt{2\lambda}} + \frac{\lambda}{27\delta_0^{\sqrt{3}}}\right) \tanh\left(\frac{\lambda^{3/2}}{\sqrt{2}\delta_0}\right). \tag{10}$$

Obviously, this choice of the function $A(\lambda, \delta_0)$ is not unique. It is also possible to choose it, e.g., as follows:

$$A = \left(\frac{\lambda}{2} + \frac{\lambda^3}{27\delta_0^{\sqrt{3}}}\right) \tanh\left(\frac{1}{\lambda^2}\right). \tag{11}$$

However, it is possible to considerably improve these formulas by means of a combination of analytical and numerical methods.

To do this, we replace in Eq. (11) the hyperbolic tangent by a Gauss hypergeometric function[12] which has a behavior qualitatively similar to that of the function $\tanh\left(1/\lambda^2\right)$ but contains some adjustable parameters which we could define via fitting the suggested function for A to the numerical data. Extensive simulations lead to the following result:

$$A = \left(\frac{\lambda}{2} + \frac{2\lambda^2}{27\delta_0^{\sqrt{3}}}\right) {}_2F_1\left(a_1, \frac{1}{2}; a_3; -\frac{\lambda^2}{\delta_0^2}\right), \tag{12}$$

where the parameters a_1 and a_2 depend on the value of the detuning parameter δ_0 only. This is the main result of the present paper.

The function $A(\lambda)$ defined by Eq. (12) is shown in Fig. 1. Examining the graph, one can conclude that there exist three distinct interaction regimes: the weak, intermediate and strong interaction regimes. In the weak interaction regime, corresponding to small values of the input parameter λ, the fitting parameter A abruptly increases reaching its maximal value at a point $\lambda = \lambda_{\max}$.

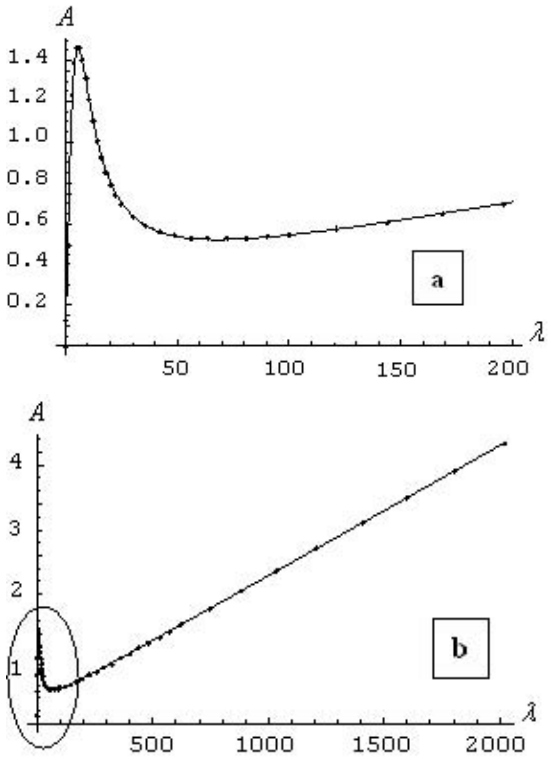


Fig. 1. Fitting parameter A as a function of λ for $\delta_0 = 5.5$. a) Weak coupling: $\sqrt{\lambda} < \delta_0$, and intermediate regimes. b) Strong coupling regime $\sqrt{\lambda} >> \delta_0$. Filled circles are the result of numerical fit while the solid line corresponds to the analytic formula 12. Figure a) corresponds to the circled region of figure b).

At sufficiently small values of λ (weak interaction *limit*) the behavior of the parameter A is given by the linear asymptote $\lambda/2$. With increasing values of λ, the parameter A starts to deviate from this linear asymptote, always remaining smaller than $\lambda/2$. Such a behavior of the parameter A is observed also in the case of the Landau-Zener model,[8] and it can be shown that this behavior is common for all the level-crossing models. Hence, we conclude that the specific character of the models is not much revealed in the weak coupling regime. For this reason, we will refer to the weak interaction limit $\lambda << 1$ as the *fundamental limit* of weak coupling regime.

Further, for $\lambda_{\max} < \lambda < \lambda_{\min}$, where $\lambda = \lambda_{\min}$ is the point where the function A has a minimum, the function $A(\lambda)$ rapidly decreases. We refer to this regime as the intermediate interaction regime. Finally, for $\lambda > \lambda_{\min}$, which corresponds to the strong interaction regime, the function A increases with λ. To gain a better insight into the behavior of the

function A in the strong interaction regime, let us consider the asymptotic expansion of the function A given by Eq. (12) at $\lambda \sim \delta_0^2$:

$$A \approx \alpha_0 \lambda + \beta_0 + \frac{\gamma_0}{\lambda}. \tag{13}$$

The structure of Eq. (12) suggests that for$\lambda >> \lambda_{\min}$ the leading asymptotic term of this expansion is determined by the product of the second term in the brackets and the hypergeometric function. Asymptotic expansion shows that for $\lambda >> \lambda_{\min}$ the slope of the term linear in λ is always positive. A further observation now concerns the structure of expansion (13). We see that in the transitional region between the weak and strong interaction regimes the function $A(\lambda)$ is approximated as a sum of the hyperbola γ_0/λ, a constant and a linear function. It is known that in the case of the Landau-Zener model the asymptotic behavior of the parameter A for $\lambda >> 1$ is given by a hyperbola only.[8] For this reason, the hyperbola in Eq. (13) can be interpreted as the Landau-Zener "imprint", while the particular character of the DK model is related to the term linear in λ. This term is mostly expressed in the strong interaction limit $\lambda >> \lambda_{\min}$ where it becomes the leading one.

Finally, we briefly discuss a peculiarity of the dependence of the final transition probability $p(+\infty)$ on the parameter λ. In the linear case, for any value of the detuning parameter δ_0 there exist values of the parameter $\lambda = \lambda_n$ such that $P_{DK}(\lambda_n, +\infty) = 1$. These values λ_n are defined from the condition $\lambda_n = \delta_0^2 + n^2/4$, where n is an integer number. In contrast to the linear case, the constructed solution suggests that in the nonlinear case, for any given δ_0 it is impossible to find a value of the parameter $\lambda = \lambda_n$ such that $p(\lambda_n, +\infty) = 1$. This property can be easily seen for large values of λ ($\lambda >> \lambda_{\min}$) when, taking into account the expansion (13), we have the following asymptotic expression for $p(+\infty)$:

$$p(+\infty) = \frac{1}{2} - \frac{1}{4}\sqrt{2\alpha_0}. \tag{14}$$

Since $\alpha_0 > 0$ we see that $p(+\infty) < \frac{1}{2}$ so that in it is impossible to convert all the atoms of the system into molecules. This limitation in the conversion efficiency can be interpreted as a saturation effect due to the nonlinearity.

3. Conclusion

Thus, we have studied the nonlinear mean-field dynamics of molecule formation at coherent photo- and magneto-association of an atomic Bose-Einstein condensate for the case when the external field configuration is defined by the quasi-linear level-crossing Demkov-Kunike model, characterized by a bell-shaped pulse and finite variation of the detuning, if the photoassociation terminology is used. Analyzing the case of *large detuning* ($\delta_0 > 1$), we have shown that there exist three different interaction regimes: the *weak coupling* (corresponding to the case when the intensity of the applied laser field is small, $\sqrt{\lambda} << \delta_0$), the opposite *strong coupling* (corresponding to the case of large field intensity, $\sqrt{\lambda} >> \delta_0$) limits, and the intermediate regime, when the detuning and the field amplitude are of the same order. A common feature of the three interaction regimes is that in all these cases the time dynamics of molecule formation is weakly-oscillatory if initially the system consists of only atomic Bose-Einstein condensate (only weak, damped oscillations between the atomic and molecular populations are observed and that occurs only after the resonance has been crossed). Using an exact third-order nonlinear differential equation for the molecular state probability, we have developed an effective variational method which has enabled us to construct a highly accurate approximate solution of the problem, which is equally applicable for the description of the association process in all the mentioned interaction regimes.

Studying the behavior of the principal fitting parameter A, we have revealed the main characteristics of the three interaction regimes. In the weak interaction regime, corresponding to small values of the input parameter λ, the fitting parameter A abruptly increases following the linear asymptote $\lambda/2$ and then deviates from it, always remaining smaller than $\lambda/2$. It can be shown that such a behavior of the parameter A is common for all the level-crossing models. Hence, we conclude that the specific character of a model is not strongly revealed in the weak coupling regime. For this reason, we refer to the weak interaction limit $\lambda << 1$ as the *fundamental limit* of weak coupling regime. Studying further the behavior of the function $A(\lambda)$ we see that there exists an interval $\lambda \in (\lambda_{\max},\ \lambda_{\min})$ where the parameter A rapidly de-

creases. We refer to this regime as the intermediate interaction regime. Finally, we see that for $\lambda > \lambda_{\min}$ which corresponds to the strong interaction regime the function A increases linearly with λ.

Further analysis of the function $A(\lambda)$ has shown that the transitional region between the weak and strong interaction limits can be approximated as a sum of the hyperbola γ_0/λ, a constant, and a function linear in λ. Comparing this decomposition with the results obtained for the Landau-Zener model we see that in the case of the Landau-Zener model the asymptotic behavior of the parameter A for $\lambda >> 1$ is given as a hyperbola only. For this reason, the hyperbola in the asymptotic expansion of A can be interpreted as the Landau-Zener "imprint", while the "true individuality" of the DK model is related to the term linear in λ. This term mostly manifests itself in the strong interaction limit $\lambda >> \lambda_{\min}$ where it becomes the leading asymptotic term.

Studying the final probability of transition to the molecular state, we have shown that within the DK model, in contrast to the linear case, in the nonlinear case it is impossible to convert all the atoms of the system into molecules. This limitation on the conversion efficiency can be interpreted as a nonlinear saturation.

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